# Correlations in hadronic wave function.

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# Motivation

Collinear factorization is "boring": all that matters is the total number of partons.

Hadrons are much more interesting than the parton model, even at not very low $p_\perp$.

In recent years we are exploring more and more structure of the hadronic wave function: TMD's, GPD's, multiple parton interactions.

Some are still pretty basic things - average distributions. Life is in the fluctuations/correlations, and we do not know much about them yet.

In particular, can it be that correlations in the initial state have something important to say about $p - A$ collisions?

This talk is about a certain type of correlations, and only within a limited framework: saturation, or CGC.

# The Ridge.

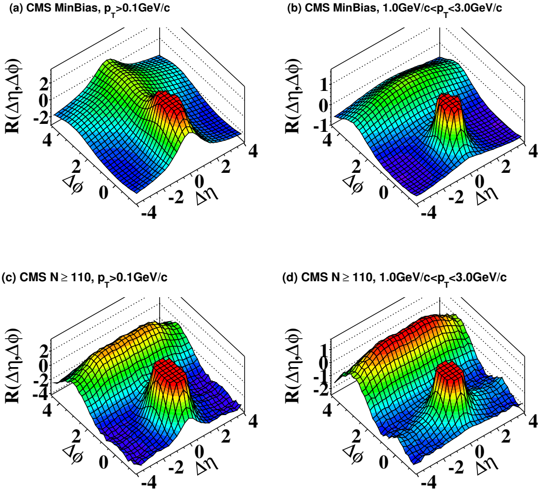



Figure: Ridge in p-p at CMS, $\sim 10^{-6}$ events

# Ridge in p-Pb.

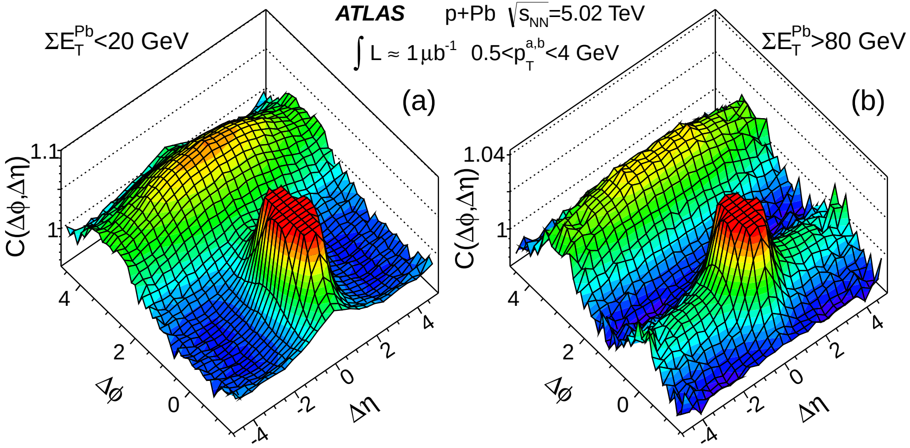
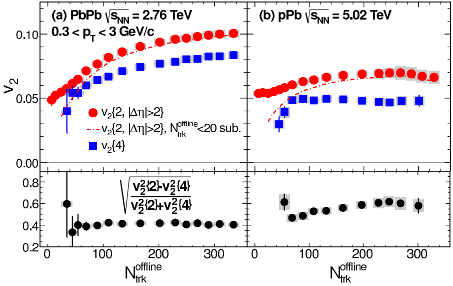
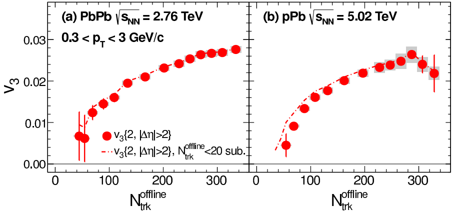



Figure: Ridge in p-Pb at ATLAS, $\sim 10^{-2}$ events

# Things got more interesting.

The correlations point to collective, or at least quasi collective behavior.



"Flow coefficients" measure correlations between the emitted particles, and are believed to encode collectivity of the final state. For double inclusive spectrum

$$\frac{d^2N}{d^2p_1 d^2p_2} = 1 + \sum_{n=1}^{\infty} 2V_n(\mathbf{p_1}, \mathbf{p_2}) \cos(n\Delta\phi)$$

$$v_n^2 = \frac{V_n(p_T, p_T^{ref})}{\sqrt{V_n(p_T^{ref}, p_T^{ref})}}; \quad n = 2, 3$$

Analogously for $v_2^4$ - from four particle inclusive spectrum.
Hydro codes seem to describe the data on $v_n$.
But: the produced system is small, the momenta involved are quite large $\sim 8Gev$, so that hydro is suspect.

Does the ridge and $v_n$ data necessarily require strong final state interactions?
**Is it possible that nontrivial initial state correlations mimic collectivity (quasi collectivity)?**

**Ridge appears in small fraction, high multiplicity events: "rare" proton configurtions with high density. Perhaps saturation is at play?**
Several possible mechanisms to generate correlations from initial state.

The one explored phenomenologically:
**"Glasma graphs"** Dumitru, Gelis, Jalilian-Marian, Lappi: Phys.Lett. B697 (2011) 21 (arXiv:1009.5295)
**Followed by qualitatively successful quantitative effort to describe data**: Dusling and Venugopalan Phys.Rev.Lett. 108 (2012) 262001 (arXiv:1201.2658); arXiv:1302.7018
In the calculation - no final state interctions. Correlations are "inherited" from the initial state.

Q: What is the physics of "Glasma graphs"?

A: Bose enhancement of gluons in the hadronic wave function.

# The CGC hadron wave function.

High energy factorisation: the fast partons are dressed by the soft gluon cloud.
Fast partons: color charge density in the transverse plane $\rho^a(x_\perp)$.
Soft gluons: the Weiszacker-Williams cloud.
Soft gluon wave function:

$$\Psi[A] = e^{i \int_{x_\perp} b_i[\rho] A_i(x_\perp)} |0\rangle$$

Solution of classical Yang-Mills equation:

$$\partial_i b_i^a(x_\perp) = g \rho^a(x_\perp)$$

$\rho$ has to be averaged over with some weight functional, e.g. simplest Gaussian: McLerran-Venugopalan model (later).

# Bose Enhancement.

Consider bosonic state with occupation numbers $n^i(p)$:

$$|\{n^i(p)\}\rangle \equiv \prod_{i,p} \frac{1}{\sqrt{n^i(p)!}} (a_i^\dagger(p))^{n^i(p)}|0\rangle, \qquad i = 1, \dots, N$$

Mean particle density:

$$n \equiv \langle a^{\dagger i}(x) a^i(x)\rangle = \sum_{i,p} n^i(p)$$

Density-density correlation?

$$D(x,y) \equiv \langle a^{\dagger i}(x) a^{\dagger j}(y) a^i(x) a^j(y)\rangle$$

Calculate in the momentum space:

$$\langle a^{\dagger i}(p) a^{\dagger j}(q) a^i(l) a^j(m)\rangle = \delta(p-l)\delta(q-m)\sum_i n^i(p)\sum_j n^j(q)$$

$$+\delta(p-m)\delta(q-l)\sum_i n^i(p) n^i(q)$$

So that:

$$D(x,y) = n^2 + \sum_i \left| \int \frac{d^3p}{(2\pi)^3}\, e^{ip(x-y)} n^i(p)\right|^2; \qquad D(p,k) = \left[\sum_i n_i(p)\right]\left[\sum_j n_j(k\right] + \delta(k-p)\sum_i (n_i(p))^2$$

# Bose Enhancement in CGC?

Bose enhancement is pretty generic, but not present in classical states.
Coherent state:

$$|b(x)\rangle \equiv \exp\{i\int d^3x\, b^i(x)(a^i(x)+a^{\dagger i}(x))\}\,|0\rangle$$

A trivial calculation gives

$$\langle b(x)|a^{\dagger i}(x)a^i(x)|b(x)\rangle = b^i(x)b^i(x)$$

$$\langle b(x)|a^{\dagger i}(x)a^{\dagger j}(y)a^i(x)a^j(y)|b(x)\rangle = b^i(x)b^i(x)b^j(y)b^j(y)$$

so

$$D(x,y) = n(x)n(y)$$

CGC is "classical fields": can they produce Bose Enhancement?
The answer is Yes.

# First things first.

Double inclusive gluon production via "Glasma Graphs":

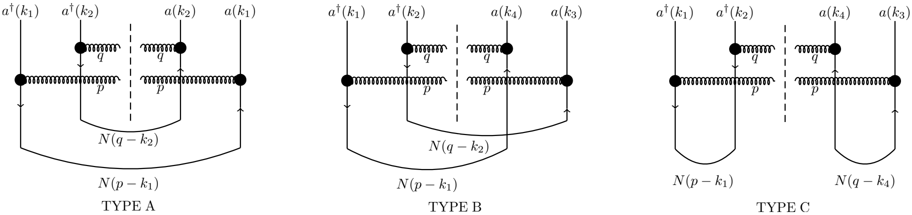



Figure: Glasma graphs for two gluon inclusive production before averaging over the incoming projectile state.

$N(k) = -\int d^2x e^{i\vec{k}\vec{x}} \langle \frac{1}{N_c} tr[S^\dagger(x)S(0)] \rangle_{\rm Target}$ - the (adjoint) dipole scattering probability.

# Gluon Production

$$\text{Type A} \propto \int_{k_1,k_2} \langle in| a^{\dagger i}_a(k_1) a^{\dagger j}_b(k_2) a^i_a(k_1) a^j_b(k_2) |in\rangle \ N(p-k_1)N(q-k_2)$$

$N(k)$ - probability of momentum transfer $k$ from the target.

IMPORTANT! $k$ - is transverse momentum only.

CGC is boost invariant: $a^i_a(k) \equiv \frac{1}{\sqrt{Y}} \int_{|\eta<Y/2|} \frac{d\eta}{2\pi} \ a^i_a(\eta, k)$

$$[a^i_a(k), a^{\dagger j}_b(p)] = (2\pi)^2 \delta_{ab} \delta^{ij} \delta^{(2)}(k-p)$$

The wave function for the soft field is classical:

$$|in\rangle_\rho = \exp\left\{ i \int_k b^i_a(k) \left[ a^{\dagger i}_a(k) + a^i_a(-k) \right] \right\} |0\rangle,$$

Weizsäcker-Williams field $b^i_a(k) = g\rho_a(k) \frac{ik^i}{k^2}$.

# The density matrix.

But the full hadronic wave function is not: one has to integrate over $\rho$ with some weight $W[\rho]$.
This defines the density matrix (operator) on the soft gluon Hilbert space:

$$\hat{\rho} = \mathcal{N} \int D[\rho]\; e^{-\int_k \frac{1}{2\mu^2(k)} \rho_a(k)\rho_a(-k)}\, e^{i\int_q b_b^i(q)\phi_b^i(-q)} |0\rangle\langle 0|\; e^{-i\int_p b_c^j(p)\phi_c^j(-p)}$$

Take MV model for $W[\rho]$:

$$\hat{\rho} = e^{-\int_k \frac{g^2\mu^2(k)}{2k^4} k^i k^j\, \phi_b^i(k)\, \phi_b^j(-k)} \left\{ \sum_{n=0}^{+\infty} \frac{1}{n!} \left[ \prod_{m=1}^{n} \int_{p_m} \frac{g^2\mu^2(p_m)}{p_m^4}\; p_m^{i_m}\, \phi_{a_m}^{i_m}(p_m) \right] |0\rangle \right.$$

$$\left. \times \langle 0| \left[ \prod_{m=1}^{n} p_m^{j_m} \phi_{a_m}^{j_m}(-p_m) \right] \right\} e^{-\int_{k'} \frac{g^2\mu^2(k')}{2k'^4} k'^{i'} k'^{j'}\, \phi_c^{i'}(k')\, \phi_c^{j'}(-k')}$$

with

$$\phi_a^i(k) = a_a^i(k) + a_a^{\dagger i}(-k)$$

# The Enhancement.

Easy to show that correlators in this $\hat{\rho}$ Wick factorize in terms of two basic elements:

$$tr[\hat{\rho} a_a^{\dagger i}(k) a_b^j(p)] = (2\pi)^2 \delta_{ab}\, \delta^{(2)}(k-p)\, g^2 \mu^2(p)\, \frac{p^i p^j}{p^4}$$

$$tr[\hat{\rho} a_a^{i}(k) a_b^j(p)] = tr[\hat{\rho} a_a^{\dagger i}(k) a_b^{\dagger j}(p)] = -(2\pi)^2 \delta_{ab}\, \delta^{(2)}(k+p)\, g^2 \mu^2(p)\, \frac{p^i p^j}{p^4}$$

So that:

$$tr[\hat{\rho} a_a^{\dagger i}(k_1) a_b^{\dagger j}(k_2) a_a^{i}(k_1) a_b^j(k_2)] = S^2 (N_c^2-1)^2 \left\{ \frac{g^4 \mu^2(k_1)\mu^2(k_2)}{k_1^2 k_2^2} \right.$$
$$\left. + \frac{1}{S(N_c^2-1)} \left[ \delta^{(2)}(k_1-k_2) + \delta^{(2)}(k_1+k_2) \right] \frac{g^4 \mu^4(k_1)}{k_1^4} \right\}$$

The first term is the "classical" square of the density.

The last term is a *bona fide* Bose enhancement contribution.

# Correlated production.

Initial state Bose enhancement $\rightarrow$ correlation in the final state.

Say projectile has saturation momentum $Q_s$, and $|k_1, k_2| \sim Q_s$: the momentum transfer in the scattering is $< Q_s$, and $N(p - k_i)$ does not have large effect.
Initial correlations are reflected in the final state (final state interactions aside!).

# Conclusions.

It's kinda interesting. Initial state correlations should be observable in some way.

Maybe it is not the ridge. But everything that may happen, will happen at some point.